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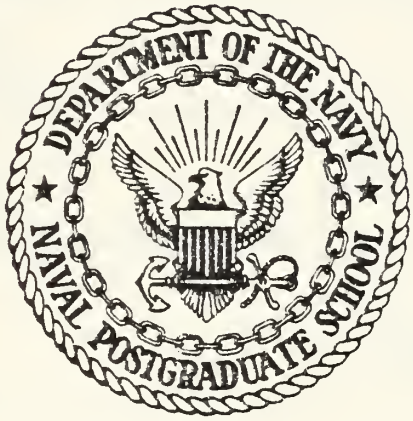


# THESIS

A MULTI-PERIOD REPAIR PARTS
INVENTORY MODEL FOR A
NAVAL AIR REWORK FACILITY

by

Andre S. Asselin

September 1983

Thesis Advisor: Alan W. McMasters

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A Ready Supply Store (RSS) containing repair parts which are anticipated to be used during the production process has been established to support the Naval Air Rework Facility (NARF). While this supporting inventory has previously been constructed using historical demand data, a single-period model and a two-period model have been proposed which compute stock levels based on quarterly production schedules. This thesis extends

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# A Multi-Period Repair Parts Inventory Model for a Naval Air Rework Facility

by

Andre S. Asselin
Commander, Supply Corps, United States Navy
B.A., College of the Holy Cross, 1969

Submitted in partial fulfillment of the
requirements for the degree of

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A Ready Supply Store (RSS) containing repair parts which are anticipated to be used during the production process has been established to support the Naval Air Rework Facility (NARF). While this supporting inventory has previously been constructed using historical demand data, a single-period model and a two-period model have been proposed which compute stock levels based on quarterly production schedules. This thesis extends the use of the projected production information in calculating RSS inventory levels from two periods to multiple periods. The disadvantage of the single-period model is that it ignores information about future schedules. The multi-period model uses the information on future schedules to behave more optimally. The multi-period model shows significant differences in inventory levels over the single-period model as a result of the added information. The multi-period model is also easily programmed on a computer and is preferred over the single-period model.

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# I. INTRODUCTION

## A. BACKGROUND

Naval Air Rework Facilities (NARFs) perform overhaul and maintenance actions on various components utilized by fleet aviation units. These rework activities are accomplished in accordance with a quarterly production schedule. Information on anticipated NARF workload requirements is also usually available for several additional future periods. The Naval Aviation Logistics Center (NALC) is in the process of developing a Material Requirements Planning (MRP) type of system which utilizes these forecasts to project requirements for individual spare parts used in the rework process. The intent is to accomplish the assigned NARF mission more efficiently by reducing work stoppages caused by stockout [Ref. 1]. The MRP system is expected to also respond better to the peaks and valleys of production and reduce surpluses and shortages which can occur when forecasting demand based on gross historical demand for an item.

MRP systems are intended to reduce or even eliminate repair part inventories by phasing item arrivals to coincide with their need within the production process. However, random variations in demand exist because an item may not always be replaced in an end item which is being overhauled. As a consequence, there is the need for the establishment of

some form of backup inventory support. The RSS will store those parts expected to be used during the production process and the Naval Supply Center (NSC) will protect those parts from issue to other NSC customers.

The MRP philosophy of inventory management mandates that these repair part stocks be carefully controlled. McMasters [Ref. 2] proposed a single-period stochastic inventory model utilizing the production information for the quarter. Hund [Ref. 3] extended that model to a two-period one with the purpose of determining how much to stock in the first quarter to meet the stochastic demands of the quarter and account for the benefits in the second quarter of any surplus from the first quarter. Both models focused only on those items which would not be replaced 100% of the time.

Hund postulated that surplus costs resulting at the end of a given period on a specific item may be greatly reduced by considering the anticipated demand for that item in future periods. A model incorporating anticipated rework activity beyond the upcoming production period should provide a more accurate reflection of total expected costs over a particular length of time, and, thus offers the potential of creating a more cost effective inventory mix. Hund did not go beyond the two-period model but did use the concept of dynamic programming to develop solutions. If a planning horizon of two periods is better than one, then an even longer horizon can be expected to be better than two.

However, it is expected that the validity of future production schedules may be questionable. In spite of this fact, a general multi-period model needs to be considered before a recommendation can be made as to what model to implement for a NARF. Questions of model complexity and data validity can then be answered through sensitivity analyses.

The basis for the development of the two-period and multi-period models is the Karlin dynamic multi-period inventory model [Ref. 4]. In that model, the demand distribution may change from period to period and the optimal policy is characterized by a single critical number representing the initial inventory value. The theorems developed by Karlin serve as useful checkpoints for operation of the two-period and multi-period inventory models and as a basis for further investigation.

## B. PURPOSE

The objectives of this thesis are to:

- Develop an inventory model for the RSS which utilizes available production schedules for as many quarters as are available;
- Obtain results from the multi-period model to compare with the single-period model or a model with only a two or three-period horizon to determine if there was any advantage to using a model with a shortened time horizon; and

- Obtain results, not addressed by Karlin in his general multi-period model analyses, for cases of decreasing production schedules and cyclic production schedules and to compare these results with the case of a constant production from quarter to quarter.

# II. THE SINGLE PERIOD MODEL

## A. INTRODUCTION

This chapter will provide a brief synopsis of the single-period model explained by McMasters [Ref. 2] and Hund [Ref. 3]. This brief description will give insight into the construction and workings of the two-period and multi-period models.

In a single-period model, the determination of how much inventory to carry at the beginning of a quarter to meet that quarter's demands for a given repair part is based solely on the production data of that one quarter. Although data may be available for subsequent quarters' production, only data from the quarter under immediate consideration can be used.

## B. GENERAL ASSUMPTIONS

The following general assumptions apply to the single-period model:

- The NARF Production schedule is known.
- The RSS inventory calculation is performed on a periodic basis once for each quarter (a periodic review model).
- Items which are certain to be replaced (100% of the time) are stocked to the needed level in the usual

sense of MRP and are excluded from application of this inventory model.

- Procurement leadtime for the quarterly stocking of the RSS to the computed levels is assumed to be zero since the supporting NSC will probably have the stock and deliveries are frequent.
- Demands in excess of RSS inventory are backordered to the NSC. It is assumed to either have the back-up stock or know where to get it quickly. A shortage penalty cost will be used to measure the cost of the inconvenience of not having a unit of the item available from the RSS.
- The probability that a part will be demanded a specified number of times during the production process is a random variable described by the binomial distribution.
- In the optimization, each quarter is treated as if it were isolated from all others.
- There is no replenishment of the RSS during the quarter to restore inventory levels for the quarter.
- The model is applied to a single repair part being used in the repair or overhaul of a single component.

C. COST ASSUMPTIONS

The following cost assumptions apply to the single-period model:

- There is no time-weighted holding cost since the space allocated to the RSS is fixed and, in general, will accommodate stocks at a high operating tempo. This space will not be decreased if inventory projections require less space. This creates a fixed cost which does not influence the model.
- Any ordering cost is ignored since the cost of each quarterly periodic review and its resultant orders would be about the same each time and essentially of a fixed nature.
- Unit cost "C" is the constant cost the NARF would pay for one unit of the item.
- Surplus cost "H" is the per unit constant penalty for having left-over repair parts at the end of the quarter's production run.
- Shortage cost "P" is the per unit constant penalty for having inadequate on-hand stock in the RSS to meet production demands. The shortage cost is a measure of the cost incurred for work stoppages due to lack of immediately available parts.

## D. THE SINGLE-PERIOD MODEL

The objective function of the model is to minimize total expected variable costs. In so doing, the model will define a unique number which will represent the optimal quantity of a repair part to be in place in RSS stock at the beginning of the NARF production quarter which is undergoing review.

The objective function of the model is represented by a total expected variable cost (TVC) equation which consists of two components. The first component is the product of unit cost of the item and the number of units proposed to be procured--denoted by "y". The second component is the sum of expected penalty costs, both shortage and surplus, at the end of the quarter based on making the same "y" quantity of the repair part as proposed to be procured in the first component available at the start of the quarter. Thus, for any given "y" a total variable cost of procuring and "carrying" that number of the item can be determined. Plotting total variable costs for each "y" against the "y" values would provide a convex curve. It is at the lowest point on the curve that the total variable cost is minimized. The associated discrete y quantity minimizes the TVC. It is denoted by "y*" since it is the optimal inventory quantity of the repair part to be carried in the RSS for the quarter in question.

Possible values of "y" range from zero to 100% of potential replacement actions. The latter coverage would result in no shortage costs and the highest expected surplus costs whereas a zero inventory would result in no surplus costs and the highest expected shortage costs. The actual demand will be somewhere between these extremes for y. The distribution of each demand is like a "Bernoulli Trial" in that it is discrete and is the result of either a "go" or "no-go"

situation. When the outcomes of these trials are accumulated over a quarter of "n" overhauls of a component containing the item in question, the total demand for the quarter is a random variable which behaves according to the binomial probability distribution.

$$g(u) = \frac{n!}{u!(n-u)!} p^u (1-p)^{n-u}; \qquad (1)$$

where:

u represents the possible demand for repair part;

p is the historical probability that a part will be replaced during the overhaul of a single component.

A recursion form of equation (1) is useful for computer calculations and is given by (2).

$$g(u) = \begin{cases} (1-p)^n \text{ for } u = 0 \\ \frac{[n-(u-1)]}{u(1-p)} pg(u-1) \text{ for } 0<u\leq n \end{cases} \qquad (2)$$

Having specified the probability function for the demand u, the sum of the expected shortage and surplus cost functions can now be written. It is

$$L(y;g) = H \sum_{u=0}^{y} (y-u)g(u) + P \sum_{u=y+1}^{n} (u-y)g(u)$$

which reduces to equation (3) when the binomial distribution is considered.

$$L(y;g) = (H+P)y \sum_{u=0}^{y-1} g(u) - (H+P) \sum_{u=0}^{y-1} ug(u) + pnP - Py \qquad (3)$$

The equation for the total variable costs for any "y" value is:

$$TVC(y) = Cy + L(y;g) \qquad (4)$$

To obtain the optimal value of y, the technique of finite differences can be used. y* will be the largest value of y for which:

$$TVC(y) - TVC(y-1) < 0$$

When this difference inequality is determined for equation (4), the result is the inequality (5). Note that $P > C$ is required to get $y > 0$.

$$\sum_{u=0}^{y-1} g(u) < \frac{P-C}{H+P} \qquad (5)$$

The minimized total variable cost can then be computed using y* in equation (4). In use, there may be a surplus of inventory available from the preceding period. If we denote this quantity by "x" then $x > 0$ if demand during the preceding period was less than the stock at the start of that period. If $x < y$ then we need to buy the difference $y-x$ so that y units will be available at the start of the quarter in question. If $x > y$ then we actually have more

than we need for that quarter and we would certainly not buy any more. We would also not need to buy any if $x = y$.

# III. THE TWO-PERIOD MODEL

## A. INTRODUCTION

This chapter will provide a brief synopsis of the two-period model developed by Hund [Ref. 3]. It builds upon the single-period model and is a specific application of the more general Karlin multi-period model [Ref. 4].

In the two-period model, the determination of how much inventory to carry at the beginning of a quarter, to meet that quarter's demands for a given repair part, is based both on the production data of that quarter and the immediately succeeding quarter. This is unlike the single-period model where production data from the succeeding quarter was not considered in determining the optimal inventory.

The reason for this analysis was to determine if knowledge of productive data from the second quarter would affect the optimal initial inventory. The comparison was performed against the single-period model results.

## B. GENERAL ASSUMPTIONS

The general assumptions of the single-period model are also valid for the two-period model except for the assumption in the optimization that each quarter is considered in isolation. In addition to the single-period general assumptions, the following are also relevant:

- The periods are successive and of equal duration (i.e., quarters of a year);
- Demand constitutes a sequence of independent random variables over successive periods.

C. COST ASSUMPTIONS

The cost assumptions of the single-period model are also valid for the two-period model. In addition, the cost functions are of the same form in each period.

D. THE MODEL

The objective function of the two-period model is the same as the single-period model--to minimize expected total variable costs. As in the single-period model, the two-period model will also generate a unique number which will represent the optimal quantity of a repair part to be in place in RSS stock at the beginning of the first NARF production quarter which is undergoing review.

The total variable cost equation of the single model will be expanded to take advantage of the additional production information from the second quarter. In keeping with the usual dynamic programming labelling of time periods, we denote the last period of a two-period sequence as period no. 1 and the first period as period no. 2.

As Hund discovered, the requirement to project the costs backwards from period no. 1 provided added complexity over the single-period model. In particular, the costs of the

period no. 1 would be affected by a balance of inventory remaining from period no. 2. That balance is a random variable, of course. The two-period model is based on the Karlin generalized inventory model [Ref. 4].

The dynamic programming technique uses inequality (5) to obtain the optimal y for period no. 1. Then it requires consideration as to whether the carry-over inventory x between period no. 2 and period no. 1 is less or more than this optimal y in the development of the expected total costs over both periods. These expected total costs are used to compute the optimal y for period no. 2.

A third component is added in the two-period model to represent the expected optimal total variable costs for period no. 1 when there is no inventory carry-over from period no. 2. This expected cost is denoted by f(0;h) for each demand situation corresponding to x = 0. Here h represents the demand distribution of period no. 1. This cost term is computed using equation (4) since x = 0, and is represented by

$$f(0;h) = TVC(k) = Ck + L(k;h) \qquad (6)$$

where

k represents the optimal y value for period no. 1 as obtained from inequality (5).

This cost from period no. 1 is then multiplied by the probability that the demand during period no. 2 will result

in x = 0. This provides the expected cost given that all the inventory was consumed in period no. 2, and that a zero carry-over into period no. 1 will then require the full period no. 2 TVC to be expended. The complete form of the third component can be written as the product:

$$f(0;h) \sum_{u=y}^{m} g(u);$$

where:

m is the production schedule of period no. 2, and

u is the possible demand value during period no. 2.

y is the inventory established at the start of period no. 2.

Since the third component represents the expected total variable cost of period no. 1 when no inventory is carried-over from period no. 2, the fourth component of the two-period model represents expected optimal total variable cost of period no. 1 when there is a positive inventory carry-over from period no. 2. This results when demand during period no. 2 is less than the unconstrained initial inventory for period no. 1. Let f(y-u;h) represent the expected optimal costs during period no. 1 given a demand $u < y$ occurred during period no. 2. Its expected value over all possible situations where $u < y$ is given by

$$\sum_{u=0}^{y-1} f(y-u;h)g(u).$$

However, there are two cases which arise for this function. One case is when the "y" value is at least as large as its counterpart second period k value; the other case is when y is less than k.

Since the application of the method of finite differences failed to provide simplifications for determining y*, Hund found it necessary to provide two equations for describing the TVC model depending on y's relationship to k. In the case where $y \geq k$, the following formula applies to describe all of the expected costs for the two-period model.

$$TVC(y) = Cy + L(y;g) + f(0;h) \sum_{u=y}^{m} g(u) +$$

$$\sum_{y=0}^{y-k} \{(H+P)(y-u) \sum_{s=0}^{y-u-1} h(s) - (H+P) \sum_{s=0}^{y-u-1} sh(s) +$$

$$pnP - P(y-u)\}g(u) + \sum_{u=y-k+1}^{m} \{(H+P)k \sum_{s=0}^{k-1} h(s) -$$

$$(H+P) \sum_{s=0}^{k-1} sh(s) + pnP + k(C-P)\}g(u) \qquad (7)$$

where:

s represents the possible demands during period no. 1.

When $y < k$, the following formula applies:

$$TVC(y) = Cy + L(y;g) + f(0;h) \sum_{u=y}^{m} g(u) +$$

$$\sum_{u=0}^{y-1} C[k-(y-u)]g(u) + \sum_{u=0}^{y-1} L(k;h)g(u). \qquad (8)$$

Hund concluded that "the critical number y* for a two-period binomial model may thus be identified by using equations (7) and (8) to compute the TVC values for all possible initial inventory quantities; it is that value of 'y' which results in the minimum TVC." [Ref. 3]

The maximum value of y is the total quantity of a given repair part needed for the quarter. This is the product of the number of units of a given repair part needed by a component and the number of components being scheduled for overhaul during the quarter. We will denote this quantity by the variable name n for period no. 2 and by m for period no. 1. The minimum value for y is, of course, zero. For the enumeration process, the y value to initiate the process can be either the maximum or zero. For the two-period model, Hund found it most efficient to begin at n and work down.

## E. TWO-PERIOD MODEL FINDINGS

The Karlin Model [Ref. 4] contains a number of conclusions which are useful checks for correctness of the multi-period model. Hund's findings which corresponded to the conclusions of the Karlin Model can be summarized as follows: [Ref. 3]

- "The critical number (y*) for period no. 2 of a two-period model is always greater than or equal to the optimal result for the corresponding one-period model...The difference in the y* values is never more than one."

- "An increasing production schedule will produce a y* value identical to the situation in which the production workload is constant."
- The shortage cost must be greater than the unit cost if the item is to be stocked at all. This was also true in the single-period model.

## F. TWO-PERIOD MODEL CONCLUSIONS

The two-period model results were compared with the single-period model results and were nearly equivalent. This lack of a substantial difference in results in view of the added complexity of the two-period model led to a recommendation that the NARF use the single-period model pending analysis of a multi-period model. The availability of workload forecasts for future periods left open a possibility that a multi-period model might provide greater benefit than a two-period model when compared to the single-period model.

# IV. THE MULTI-PERIOD MODEL

## A. INTRODUCTION

This chapter will describe the composition of a multi-period model. It is a general finite period model which builds upon the two-period model of Hund and the general Karlin multi-period model.

In the multi-period model, the determination of the optimal inventory to carry at the beginning of a sequence of t periods is based on the production data of that period and as many successive periods as the user desires to include based on available production data. The optimal inventory is determined using dynamic programming. This model, unlike the single-period and the two-period models, allows the user to make use of as much quarterly production data as is available. The user sets the time horizon (for example, eight quarters) and then steps forward one quarter moving the same horizon ahead one quarter. The principle for optimality from dynamic programming states that:

> "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision." [Ref. 5]

There were three goals for developing the multi-period binomial distribution inventory model. The first was to develop an inventory model for the RSS which utilizes

available production schedules for as many quarters as are available. The second goal was to obtain results from the multi-period model to compare with the single-period model or a model with only a two or three-periods horizon to determine if there was any advantage to using a model with a shortened time horizon. The third goal was to obtain results, not addressed by Karlin in his general multi-period model analyses [Ref. 4], for cases of decreasing production schedules and cyclic production schedules and to compare these results with the case of a constant production from quarter to quarter.

## B. ASSUMPTIONS

All general assumptions and cost assumptions associated with the two-period model of chapter III apply to the multi-period model. The general model adds the additional freedom of allowing future periods to be discounted.

## C. THE MULTI-PERIOD MODEL

The objective function of the multi-period model is the same as the single-period and two period models--to minimize expected total variable costs for the selected time horizon. As in the previous models, the multi-period model will also generate a unique number which will represent the optimal quantity of a repair part to be in place in RSS stock at the beginning of the first NARF production quarter, period no. t, of the time horizon. The various quarters of

the time horizon are numbered in inverse chronological sequence. The need for this inversion will be discussed later. A subscript "t" will be used to denote the quarter. As an example, when t=1, $y_t^*=k$ where k was the optimal y value for period no. 1 and for the single-period model.

The cost components of the multi-period objective function are identical to those of the two-period model in concept. The primary difference is in the third and fourth components. When $y_t \leq y_{t-1}^*$, the expected total variable cost equation for the multi-period model can be written as

$$TVC(y_t) = Cy_t + L(y_t;g_t) + f(0;g...) \sum_{u=y_t}^{\max n} g_t(u) + \sum_{u=0}^{y_t-1} f(y_t-u;g...)g_t(u); \qquad (9)$$

where:

$f(x;g...)=f(x;g_{t-1},...,g_1)$ when x is the carry-over from the prior period;

max n represents the highest planned production schedule of all quarters;

t=number of the period in inverse chronological sequence.

For the case when $y_t > y_{t-1}^*$, the expected total variable cost equation is:

$$TVC(y_t) = Cy_t + L(y_t;g_t) + f(0;g...) \sum_{u=y_t}^{\max n} g_t(u)$$

$$+ \sum_{u=0}^{y_t-y^*_{t-1}-1} f(y_t-u;g...)g_t(u)$$

$$+ \sum_{u=y_t-y^*_{t-1}}^{y_t-1} f(y_t-u;g...)g_t(u). \qquad (10)$$

At each periodic review, the calculation of the beginning inventory balance (for period no. t) is performed assuming adherence to the principle of optimality of dynamic programming. This assures consistent application of the model at each review. During each review, the overall problem of finding the optimal expected total variable cost based on data from multiple quarters is broken into as many sub-problems as there are quarters. Also, approximate recurrence relations for the t period case (where period t comes first in time) are used in functional equations as a means of simplifying the solution to the overall problem.

The terms f(0,g) and f(y-u,g) can be generally denoted f(x,g) where x is the carry-over. Following Karlin, f(x,g) represents the optimal TVC of an immediately succeeding quarter given that x is the carry-over value from the quarter before. In the case of the third component of the TVC equation, the carry-over is zero and the succeeding period's optimal TVC value is represented as f(0,g).

The fourth component of the TVC equation represents the optimal TVC of the immediately succeeding period when the

value of carry-over is less than $y_t^*$. The equation for $f(x;g_t)$ is given by (11)

$$f(x;g...) = TVC(y_{t-1}^*) - Cx \text{ for } 0 \leq x \leq y_{t-1}^*; \quad (11)$$

where:

Cx represents the value of the carry-over which does not have to be procured in the succeeding quarter.

When the carry-over exceeds $y_t^*$, then the optimal TVC for the succeeding period is constrained to $y_t = x$. This is represented by (12).

$$f(x;g...) = TVC(y_{t-1}=x) - Cx \text{ for } x > y_{t-1}^*. \quad (12)$$

The iterative application of equations (9) and (10), as appropriate, through each of the periods from period one thru period eight would yield an optimal TVC and its associated optimal initial inventory value which is to be carried in the first chronological period. In the process, the optimal inventory values for all subsequent periods are also computed. This process would be repeated for each item in the RSS inventory which is not stocked to meet a 100% replacement policy. The entire periodic review for the next quarter's production support inventory need not be made until a quarter later. No intervening computations are required. The earlier computed optimal inventory values could be used for stocking for those periods or the whole model could be re-run. A complete re-run has the advantage

of using firmer forecasts of the workload. In order to facilitate understanding, the process has been put into flowchart form in Appendix A.

## D. PRESENT VALUE CONSIDERATION

The multi-period model has the capability for considering the effect of carry-over penalties for a considerable time in the future. Since the objective function is to minimize the expected total variable cost of the first chronological period, assuming optimal inventories are stocked for all periods, the dollar streams for all periods should be represented in terms of their present value. The optimal expected cost values of the future chronological periods are contained in the f(x,g) values. Discounting these f(x,g) values by a constant factor will therefore provide the compounding required for all the future periods. The selected discount factor should fit the duration of the period to achieve the annual discount desired. The factor is represented by "A." The location of the discount factor for the conditions of equation (9) is shown in equation (13). The location of the discount factor for the conditions of equation (10) can be deduced from the comparison between (13) and (9).

$$TVC(y_t) = Cy_t + L(y_t;g_t) + A[f(0;g...) \sum_{u=y_t}^{max\ n} g_t(u) + \sum_{u=0}^{y_t-1} f(y_t-u;g...)g_t(u)] \quad (13)$$

If the desired annual discount is 10% then A=1 - 0.10 = 0.9 for the present value of the first year's costs. The corresponding quarterly A factor would then be the fourth root of 0.9 or .974.

## E. MULTI-PERIOD MODEL FINDINGS

This chapter can be summarized as follows:

- The Karlin multi-period model can be easily adapted to use with the discrete binomial distribution.
- The multi-period model requires an iterative solution procedure based on dynamic programming principles because of the discrete nature of the binomial distribution.
- The multi-period model determines the optimal expected total variable cost over any number of periods and identifies the associated optimal initial inventory quantity.

# V. MULTI-PERIOD SENSITIVITY ANALYSIS

## A. DISCUSSION

As was stated in chapter IV, there are three goals for the multi-period model. The first was to develop an inventory model for the RSS which utilizes available production schedules for as many quarters as are available. The second goal was to obtain results from the multi-period model to compare with the single-period model or a model with only a two or three-periods horizon to determine if there was any advantage to using a model with a shortened time horizon. For the latter goal, the basis of the comparison is the value of the optimal inventory level. The third goal was to obtain results, not addressed by Karlin in his general multi-period model analyses, for cases of decreasing production schedules and cyclic production schedules and to compare these results with the case of a constant production from quarter to quarter. Important to all of these goals is the behavior of the model results under changing parameter values. This sensitivity analysis will include variations in the overhaul schedule, the unit procurement cost, the shortage cost, the surplus cost, and the probability of a repair being needed for overhaul of a component. Unit, shortage, and surplus costs are variable as well as the

probability of the need for repair of a part, and the discount factor.

B. VARYING PRODUCTION SITUATIONS

In order to conduct the sensitivity analyses, a computer program (see Appendix B) was written to carry out the steps of the flow diagram in Appendix A. This program can be used for any finite number of periods. To address the first goal, optimal results were obtained for decreasing and varying production schedules as well as steady state (no change in production schedule from period to period) and increasing production schedules. The time horizon was eight periods, or two years, corresponding to the maximum expected planning horizon for a NARF.

A range of quarterly production values from zero up to thirty were selected corresponding to engine overhaul schedules examined by Slaybaugh [Ref. 1]. A schedule of fifteen was used as the starting value (period no. 8) in studying the effect of variations.

In determining the parameter values, we have previously assumed that the surplus cost should not exceed unit cost since unit cost would be the highest credit allowed by the supply system if the excess were returned for credit. The shortage cost should be greater than the unit cost otherwise, as Hund determined, the item will never be stocked since it would be cheaper to be out of stock. Therefore, as

Hund also determined, unit cost (C) will be selected between H and P.

Eleven different possible cases were examined and are explained as follows:

- Case 1: the "steady state" production situation where production for any quarter is the same number of components as for every other quarter. In this case the following production numbers were used: n8=15, n7=15, n6=15, n5=15, n4=15, n3=15, n2=15, n1=15.
- Case 2: the increasing production situation where the production in each successive quarter increases. The following production numbers were used: n8=15, n7=17, n6=20, n5=22, n4=24, n3=26, n2=28, n1=30.
- Case 3: the decreasing production situation where the production in each successive quarter decreases. The following production numbers were used: n8=15, n7=12, n6=10, n5=8, n4=6, n3=4, n2=2, n1=0.
- Case 4: the cyclic production situation where production erratically goes from the median to the maximum then to the median and down to the maximum then to the median and down to the minimum. The following numbers were used: n8=15, n7=30, n6=15, n5=0, n4=15, n3=30, n2=15, n1=0.
- Case 5: the cyclic production situation where production erratically behaves the opposite from case 4 in the equivalent polar time periods. The following

production numbers were used: n8=15, n7=0, n6=15, n5=30, n4=15, n3=0, n2=15, n1=30.

- Case 6: the immediate increase to maximum production situation where in period no. 7 production increases to the maximum allowed and remains at the maximum for the remainder of the periods. The following production numbers were used: n8=15, n7=30, n6=30, n5=30, n4=30, n3=30, n2=30, n1=30.
- Case 7: the production termination situation where production immediately goes to zero after the first quarter. The following production numbers were used: n8=15, n7=0, n6=0, n5=0, n4=0, n3=0, n2=0, n1=0.
- Case 8: The decrease to the minimum production situation where production immediately goes to the lowest level of one per quarter and remains there. The following production numbers were used: n8=15, n7=1, n6=1, n5=1, n4=1, n3=1, n2=1, n1=1.
- Case 9: The modified two-period equivalent to Case 7 where there is decreasing production from the first to the second quarter and then no production thereafter. The following production numbers were used: n8=15, n7=5, n6=0, n5=0, n4=0, n3=0, n2=0, n1=0.
- Case 10: The modified two-period equivalent to Case 8 where production goes to the lowest level of one per quarter and remains there beginning with the third

quarter. The following production numbers were used: n8=15, n7=5, n6=1, n5=1, n4=1, n3=1, n2=1, n1=1.

- Case 11: The modified three-period equivalent to Case 7 where there is decreasing production from the first to the second to the third quarters and then no production thereafter. The following production numbers were used: n8=15, n7=10, n6=5, n5=0, n4=0, n3=0, n2=0, n1=0.

The fluctuations in the production numbers of some cases, to the "poles," provided worst-case scenarios for comparison with steady-state.

For each of the cases, a number of different values of the cost parameters were tested to determine how their variations affect the results. Ten sets of cost parameter values were applied to each case. This "matrix" was then evaluated for three different values of the probability of the part requiring replacement. Probabilities of 0.1, 0.5, and 0.9 were used. A final sensitivity analysis was conducted using the steady-state (Case 1) situation without a discount factor in order to determine the impact, if any, of the annual 10% discount applied on the matrices.

The results of the sensitivity analysis are presented in Appendix C.

## C. RESULTS

Karlin theorized that an increasing production schedule would result in the initial inventory requirement being the

same as the steady-state production schedule (see Case 1). Case 2 is the situation of the increasing schedule and agrees with the Karlin findings. Surprisingly the varying production situations in Cases 4 and 6 where production goes up and down and increases to the maximum respectively, behave in a manner consistently similar to the increasing situation. The only explanation is the dominating effect of increased production in period no. 7 on the final results.

In case 3, the gradually decreasing production situation, the y* values remained almost identical to the steady-state situation. In one instance the y* value went down by one from the steady-state. In the reverse situations of Cases 2 and 4, the down-and-up situation (Case 5) was either the same as the steady-state or one less and the decrease to the minimum situation (Case 8) was either one or two less than steady state except when y* was zero or when the probability was 0.9. The situation in Case 3 seems to indicate that a slow and gradual decrease allows the y* value to closely correspond to the steady-state situation. However, the more wildly fluctuating situations create a wider gap. Case 10 forces the production decreases to the minimum of one by the third quarter. When the probability value was 0.1 the results were still no more than two units below the steady-state. When the probability value was 0.5, the results were within 1 and, when the probability value was 0.9, the results matched the steady-state. Therefore,

depending on the probability, this range of numbers showed no greater difference than two from the steady-state.

If production were to terminate entirely in the second, third, or fourth quarter the results are different than if some minimum level is sustained. In Case 7 where production ceases immediately after the first quarter, the y* values were further from the steady-state situation the lower the probability and for a given probability it was lower for the lower shortage cost. In Case 9 where the second quarter had production and then no production in succeeding quarters, the 0.1 and 0.5 probability situations were within two of steady state whereas the 0.9 matrix was a match to steady-state. In addition, this case matched Case 8 (decrease to the minimum) except in one instance where it was one higher. In Case 11 where the second and third quarters had decreasing production and then no production in succeeding quarters, the 0.1 probability matrix stays within two below the steady-state, and the 0.5 and 0.9 matrices match the steady-state.

As another means of comparison, Case 7 was compared to the single-period model, Case 9 to a two-period model, and Case 11 to a three-period model. The two and three-period model results are in Appendix D. The eight-period y* values are the same as or less than the respective shorter period models. Again, the probability was a significant factor when deviations occurred, however the shortage cost was not an

influencing element in the differences. The three-period model was the most closely in line with the eight period results. Therefore it appears that a planning horizon can be shorter as the probability value increases. Additionally, the accuracy of the production schedule is not so serious several periods downstream.

In general, when the discount factor is ignored the results are the same as in using a ten-percent discount except when there is no surplus cost. When the probability was 0.5 and 0.9, the y* value surged to the maximum without a surplus cost. When the probability was 0.1, the y* values increased but not to the maximum for the state when there was no surplus cost. There were occasional unpredictable variances of one in the other states of cost parameters.

## D. COMPARISONS WITH THE SINGLE-PERIOD MODEL

Hund found the two-period model to provide some minor improvements and a lower total variable cost than the single-period model [Ref. 3]. The improvements, however, were more than offset by the added complexity. As a result of the increased complexity, the single-period model was favored over the two-period model. The question left unanswered was whether or not the minor improvements gained by use of the two-period model would be magnified in a multi-period model and that the benefits would then exceed any added complexity in a multi-period model.

Appendix E contains the results of the single-period model when n equals 15 and the probability is 0.1, 0.5, or 0.9. In general, the single-period model results are less than or equal to any multi-period model which behaves like the steadily increasing or steady state situations. They are also more than or equal to the decreasing state multi-period model results. The multi-period model is responding to information concerning future period requirements--information not used in the single-period model.

The probability, of a part being required to be replaced, is again of considerable importance as are the penalty costs. When the probability of replacement is high (0.9), the difference between the single-period model and the multi-period model results, under any situation, is never greater than one. As the probability decreases, the frequency and magnitude of any differences increases. The differences almost disappear as the surplus cost approaches the unit cost or as the shortage cost increases significantly in relation to the unit cost. The greatest difference occurs for the single period when C=250, H=0, P=1000, the probability is 0.1, n=15 for both the 8-period steady-state and for the single-period models.

# VI. MULTI-PERIOD MODEL CONCLUSIONS

## A. GOALS RESTATED

As noted earlier, there were three goals for the multi-period model. The first was to develop an inventory model for the RSS which utilizes available production schedules for as many quarters as are available. The second goal was to obtain results from the multi-period model to compare with the single-period model or a model with only a two or three-period horizon to determine if there was any advantage to using a model with a shortened time horizon. The third goal was to obtain results, not addressed by Karlin in his general multi-period model analyses [Ref. 4], for cases of decreasing production schedules and cyclic production schedules and to compare these results with the case of a constant production from quarter to quarter.

## B. STEADY-STATE COMPARISON CONCLUSIONS

The following are general conclusions drawn from the analyses of chapter V concerning the decreasing and varying production situations when compared with the steady-state production situation:

- Whenever there is an increase in production from the first chronological period to the second period, the results will follow the constantly increasing

production situation and be the same as the steady-state production. (There is a borderline situation where this did not prove true. When there was no discount applied and C=250, H=0, p=.1, and P=1000 or 10,000, the increasing case provided a higher inventory than the steady-state case. No known reason could be determined for the situation and further investigation is recommended.)

- Gradually decreasing production situations will be the same or minus one from the steady-state production situation.
- Fluctuating production situations where the second period production decreases from the first period behave like the gradually decreasing situation.
- As long as succeeding periods show some production, the y* values will be within two of the steady-state situation sustaining the first period's production.
- A shorter planning horizon will have y* values which are less than the eight quarters' steady-state result. The magnitude of the probability that the part will require replacement has a strong influence on the y value.

C. SINGLE-PERIOD COMPARISON CONCLUSIONS:

The following are general conclusions drawn from the analysis of chapter V concerning the comparison of the multi-period results with single-period results:

- For multi-period steady-state, gradually decreasing, or increasing production schedules, the y* value for the multi-period model exceeds or is equal to the single-period results.
- For multi-period situations where there is a rapid decrease from the first chronological period to the second or when production levels out at the minimum or terminates, the y* value for the multi-period model is less than or equal to the single-period model.
- The penalty cost values are significant determinants of the magnitude of difference between the single-period and the multi-period results. The lower the penalty costs, the larger the difference.
- The probability that a part will require replacement is a significant determinant in how close a multi-period situation will match the single-period results. The higher the probability, the closer the y* values correspond in the single period and multi-period models.
- The total variable cost (TVC) of the multi-period model will exceed the single-period TVC for the equivalent periods when all quarters production equal or exceeds the single-period production.

## D. RECOMMENDATIONS

In his dynamic multi-period inventory model, Karlin was able to state that for increasing production functions, the multi-period model would yield a critical number which was the same as the steady state situation. No similar corollary is apparent for the decreasing production situation. However, it is clear that the multi-period model is more responsive to future period information.

There is an apparent benefit in using the multi-period model based on the more realistic consideration of holding inventories for periods in which it may never be used. The planning horizon of the multi-period model adjusts downward for future production downturns. It is most important to correctly predict the production trend as to whether it is increasing, decreasing, or cyclic and that the first two or three periods be as accurate as possible. The downstream numbers need not be as accurate as long as they correctly depict the trend.

Additionally, if all periods' data is accurate at the outset and remains unchanged, then there is no need for future inventory calculations since the model predicts the optimal level for each quarter.

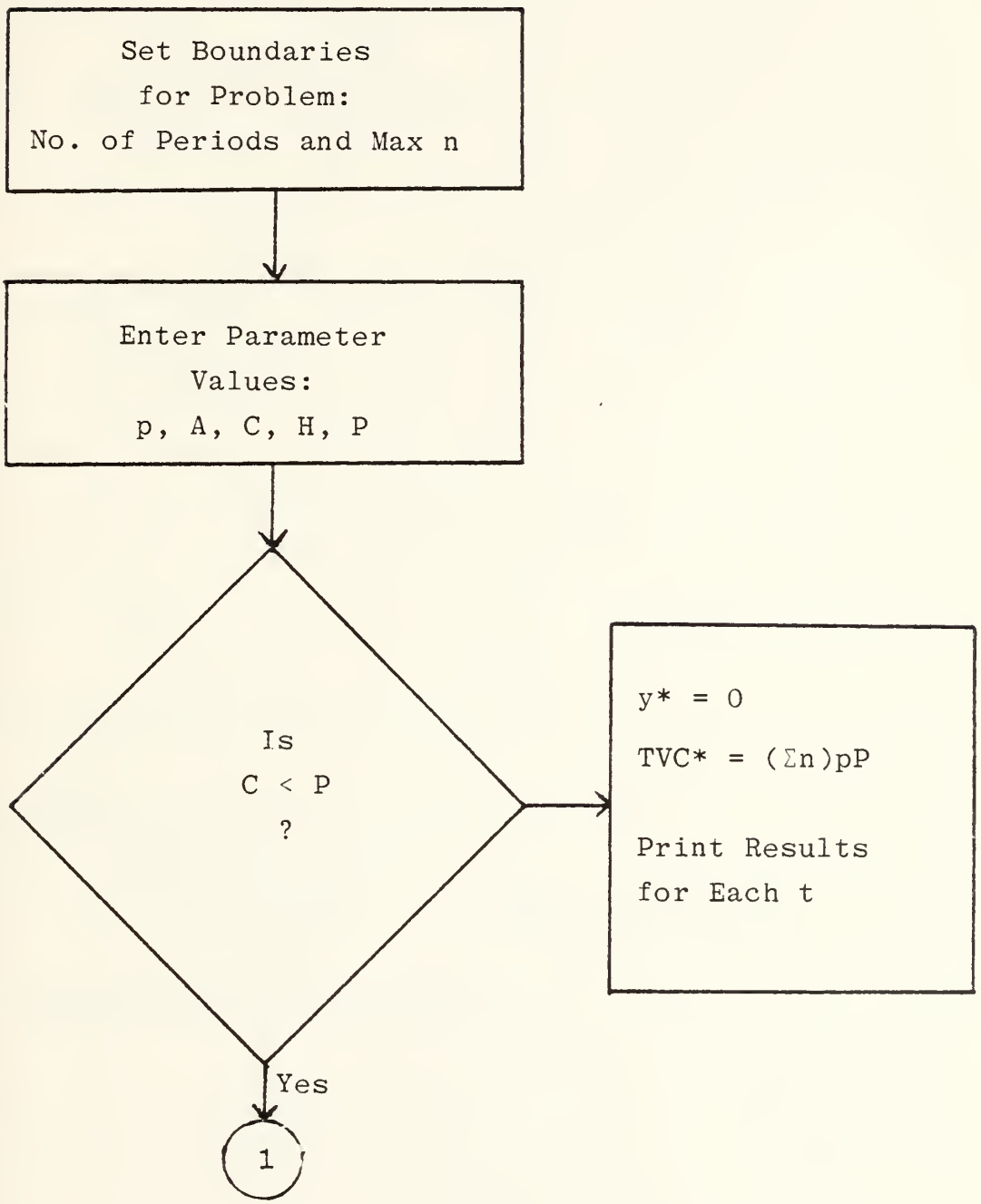
The problem of obtaining an accurate estimate of probability for part replacement and an accurate shortage cost would be present in any model. Both models assume that any surplus would be returned immediately at the end of a

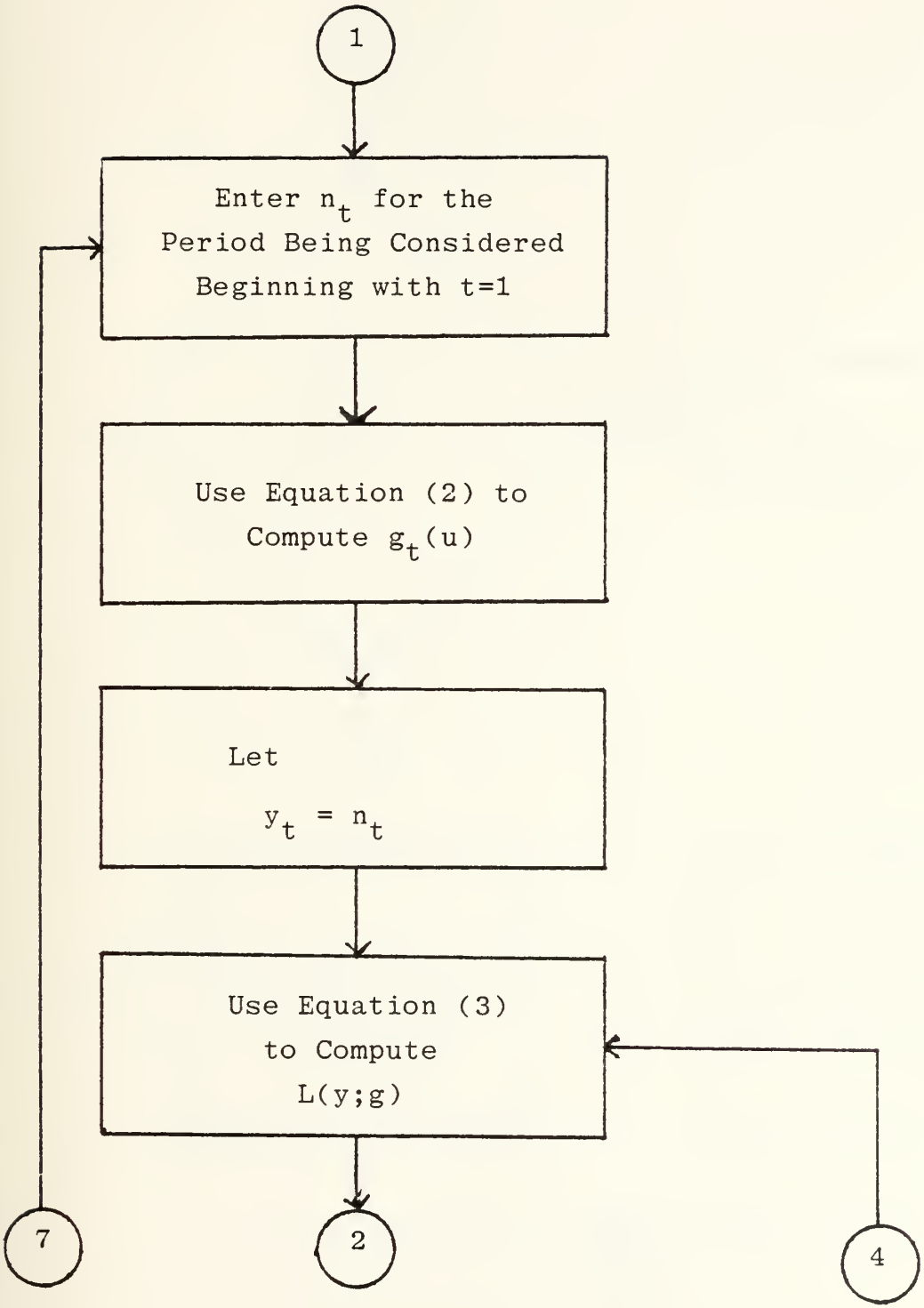
quarter and the credit allowed for the turn-in is the basis for the surplus penalty cost.

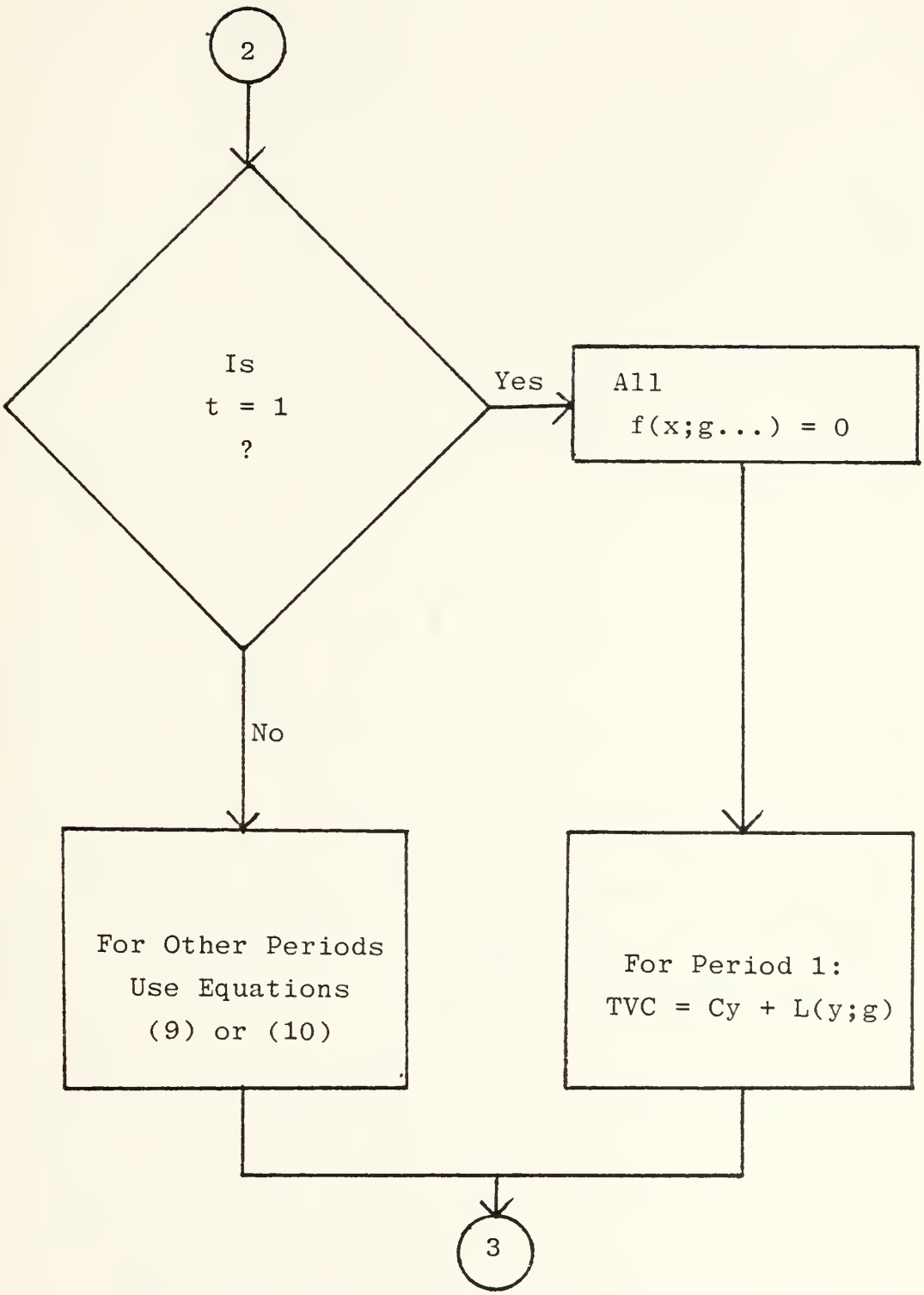
The multi-period model has proven relatively easy to develop and use. It is a model which will provide the user with a degree of flexibility in selecting the planning horizons. Therefore, it is recommended that the single-period model be replaced by the multi-period model.

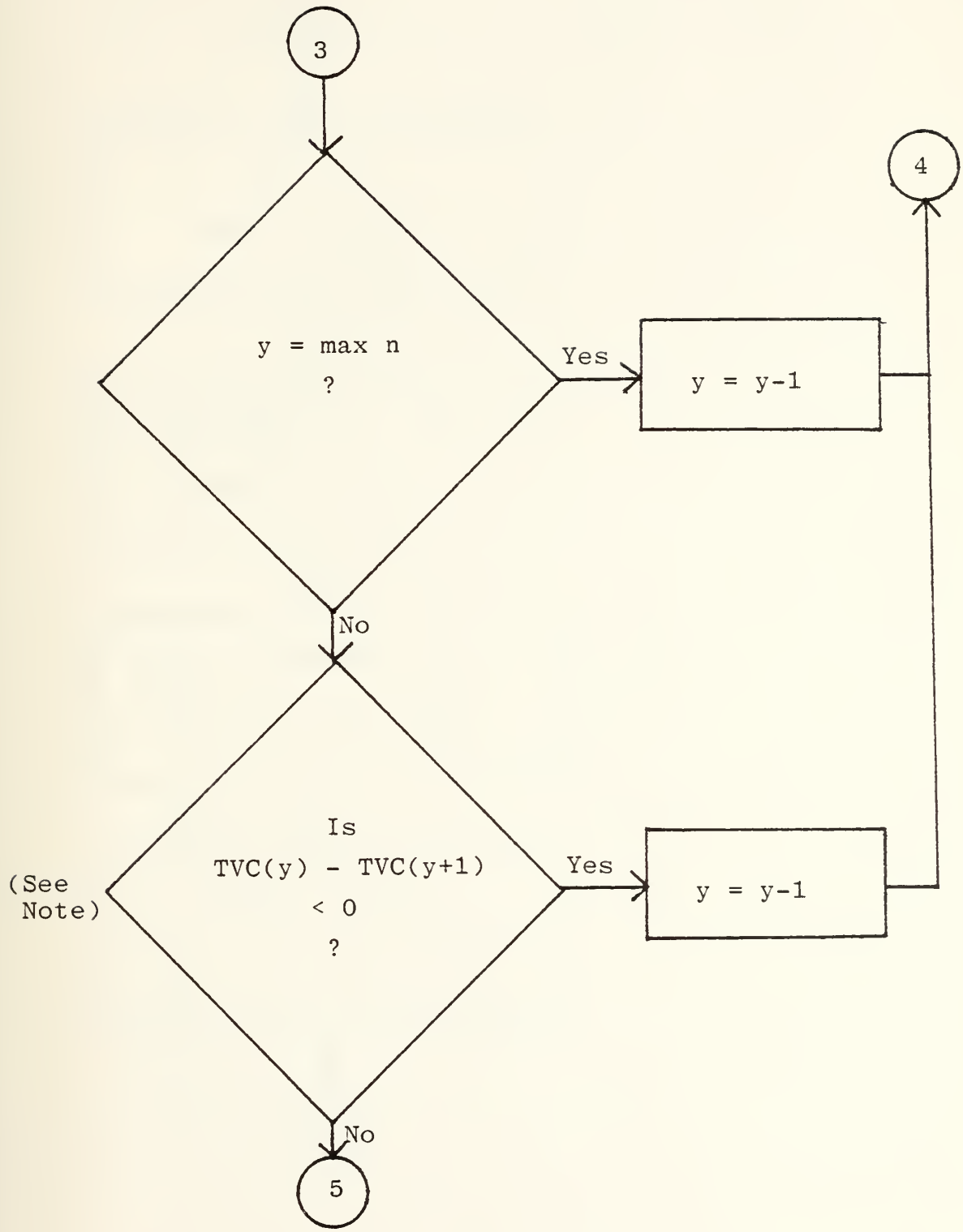
APPENDIX A

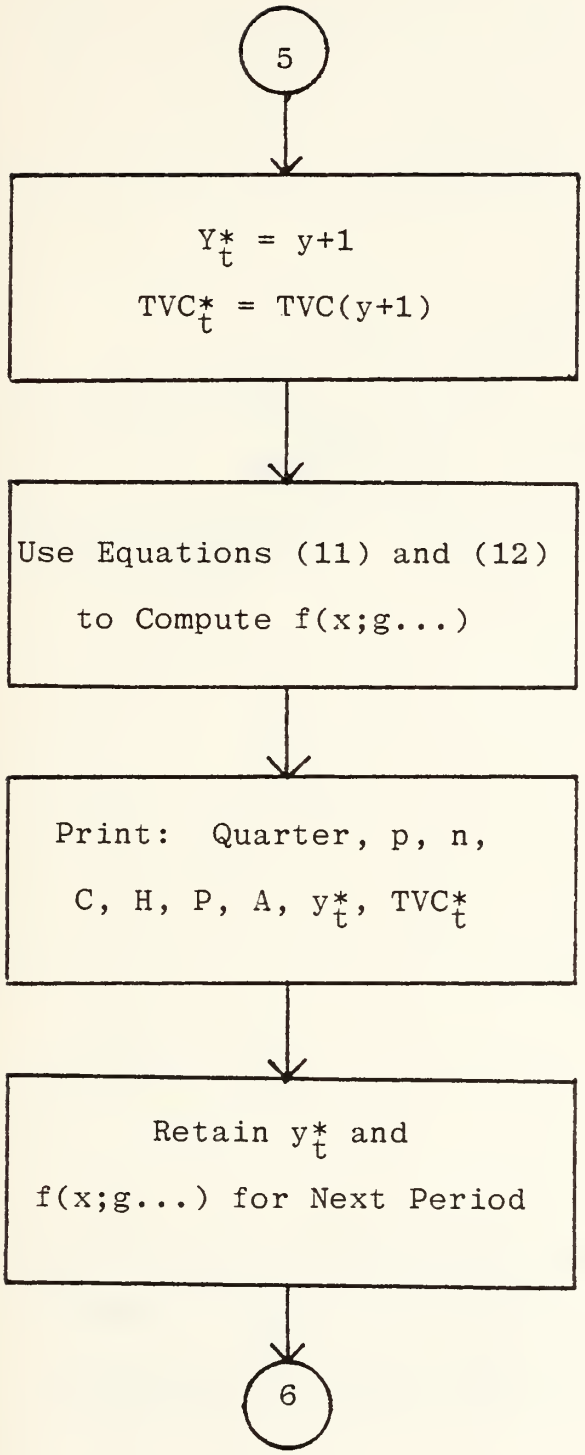
MULTI-PERIOD INVENTORY MODEL FLOWCHART

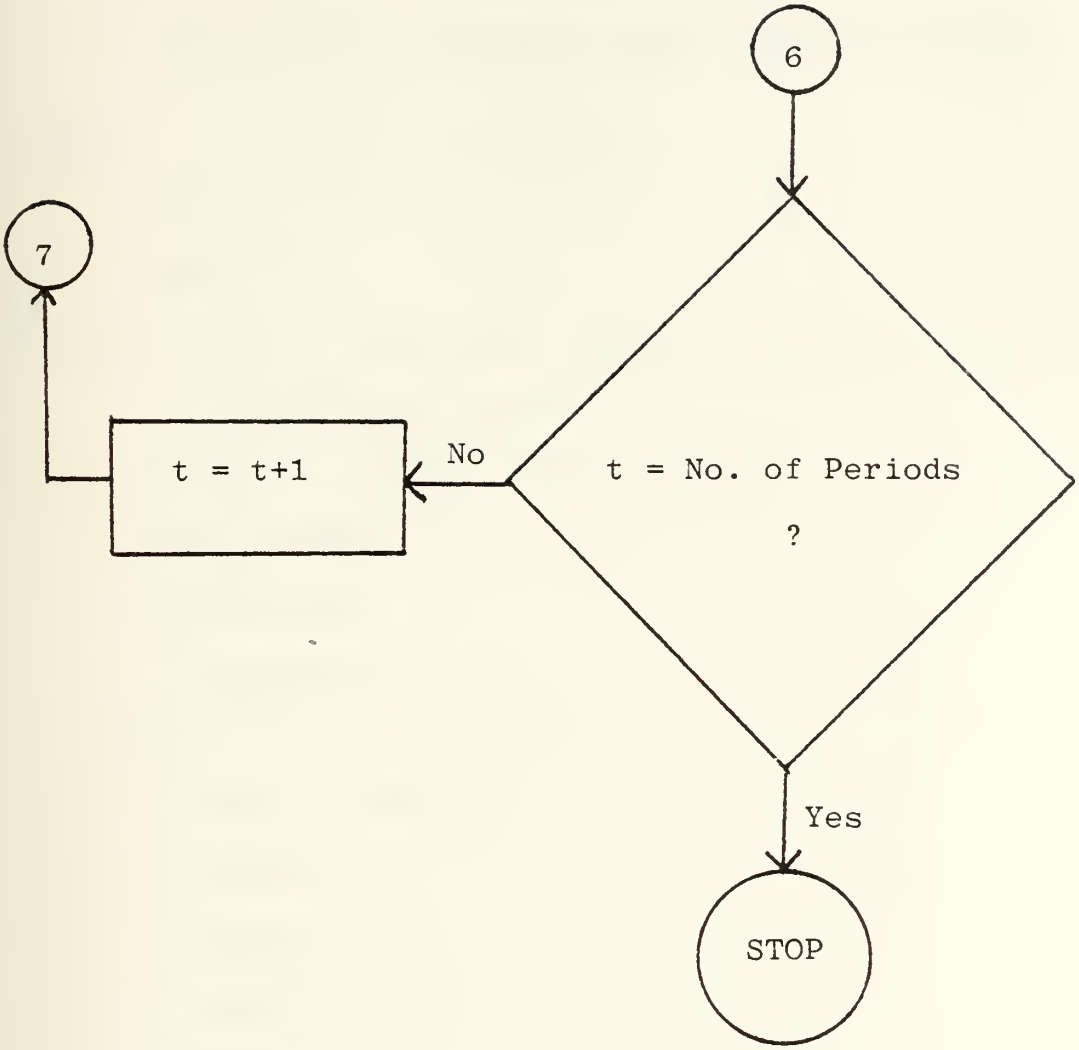



1
Enter $n_t$ for the Period Being Considered Beginning with t=1
Use Equation (2) to Compute $g_t(u)$
Let $y_t = n_t$
Use Equation (3) to Compute L(y;g)
7
2
4


2
Is
t = 1
?
Yes
All
f(x;g...) = 0
No
For Other Periods
Use Equations
(9) or (10)
For Period 1:
TVC = Cy + L(y;g)
3


3
y = max n
?
Yes
y = y-1
4
No
Is
TVC(y) - TVC(y+1)
< 0
?
(See
Note)
Yes
y = y-1
No
5


5
$Y_t^* = y+1$
$TVC_t^* = TVC(y+1)$
Use Equations (11) and (12)
to Compute f(x;g...)
Print: Quarter, p, n,
C, H, P, A, $y_t^*$, $TVC_t^*$
Retain $y_t^*$ and
f(x;g...) for Next Period
6




Note: To find lower boundary for any possible multiple optimal situations, change the strict inequality of "less than" to "less than or equal to."

## APPENDIX B

## MULTI-PERIOD INVENTORY MODEL COMPUTER PROGRAM

```
$JOB
      WRITE(6,2)
      READ(8,*) A
C  *******'A' IS FOR ALPHA WHICH IS THE FOURTH ROOT OF
          (1-DISCOUNT FACTOR)
2     FORMAT(' ','ENTER ALPHA FACTOR')
      PRINT,'ALPHA=',A
      DO 9 I=1,10
         WRITE(6,3)
C  *******NOTE:  PROB=p******
         READ(8,*)PROB
         WRITE(6,4)
         READ(8,*)C
         WRITE(6,5)
         READ(8,*)H
         WRITE(6,6)
         READ(8,*) P
3        FORMAT('0','ENTER PROB')
4        FORMAT('0','ENTER C')
5        FORMAT('0','ENTER H')
6        FORMAT('0','ENTER P')
         CALL MCTWO(LIM,N,A,PROB,H,P,C,TVCSTR,KYSTR,KPERIO)
```

```
9     CONTINUE
      STOP
      END
      SUBROUTINE MCTWO(LIM,N,A,PROB,H,P,C,TVCSTR,KYSTR,KPERIO)
1     DIMENSION QUARTR(8),KYSTAR(8),G(31),TVC(31),F(31)
C     **REMEMBER**IF 'N' CHANGES, THEN SO DO 1,2*******
C
C***** RESET THE INPUT DATA FILE FROM LAST ITERATION *****
      REWIND 9
C
C****** DEFINE THE NUMBER OF QUARTERS *********
      LIM = 8
C*********************************************
C
      IF(C.GE.P) GO TO 950
      NMAX = 30
      I=1
      II=0
      KPERIO=0
10    KPERIO=KPERIO+1
      II=II+1
      KY=NMAX+2
      READ(9,*) N
C     CALCULATE THE PROBABILITY THAT N ITEMS WILL BE DEMANDED
C     VECTORS CANNOT OPERATE WITH "0",SO G(1)=P(0),G(2)=P(1),ETC
      NPLUS1=N+1
```

```
      Q=1-PROB
      DO 20 I-1,NPLUS1
         IF(I.GT.1) GO TO 21
         G(1)=Q**N
         GO TO 20
21       G(I)=G(I-1)*PROB*(N-(I-2))/((I-1)*Q)
20    CONTINUE
      NM=NMAX+1
      NPLUS2=N+2
      IF(NPLUS2.GT.NM)GO TO 30
      DO 22 I=NPLUS2,NM
22    G(I)=0
30    KY=KY-1
      KYMIN1=KY-1
C  CALCULATE L(Y,G) FOR ANY PERIOD (TPENAL)
      GYSUM=0.0
      GSUMU=0.0
      IF (KYMIN1.LT.1.0) GO TO 24
      DO 23 I=1,KYMIN1
         GYSUM=GYSUM+G(I)
23    GSUMU=GSUMU+(I-1)*G(I)
      PENY=(H+P)*GYSUM*KYMIN1
      PENU=(H+P)*GSUMU
      TPENAL=PENY-PENU+(PROB*N*P)-(P*KYMIN1)
      GO TO 25
24    TPENAL=PROB*N*P
```

```
25     GFSUM=0
       GXSUM1=0
       GXSUM2=0
       ZEROX=0
       IF (KPERIO.EQ.1 ) GO TO 800
C CALCULATE F(0,G) ,(ZEROX) , FOR OTHER THAN PERIOD 1
       GNSUM=0.0
       IF(KY.LT.1.0) GO TO 51
       DO 50 I=KY,NM
          GNSUM=GNSUM+G(I)
50     CONTINUE
       GO TO 53
51     DO 52 I=1,NM
          GNSUM=GNSUM+G(I)
52     CONTINUE
53     ZEROX=F(1)*GNSUM
C DECIDE IF CASE 1 (Y2.LE.Y1*) OR CASE 2 (Y2.GT.Y1*)
       W=KY-KYSTR
       IF(W.GT.0.0) GO TO 400
C      CASE 1 ,Y2 LE Y1*
       GFSUM=0.0
       IF(KYMIN1.LT.1.0) GO TO 101
       DO 100 I=1,KYMIN1
          GFSUM=GFSUM+G(I)*F(KY-(I-1))
```

```
100   CONTINUE
      GO TO 102
101   GFSUM=0
102   GXSUM1=0
      GXSUM2=0
      GO TO 800
C     CASE 2,Y2 GT Y1*
400   KZ=KY-KYSTR-1+1
      GXSUM1=0.0
      KZK=KZ+1
      IF(KZ.LT.1.0)GO TO 406
      DO 405 I=1,KZ
         GXSUM1=GXSUM1+G(I)*F(KY-(I-1))
405   CONTINUE
406   GXSUM2=0.0
      IF (KYMIN1.LT.KZK)GO TO 451
      DO 450 I=KZK,KYMIN1
         GXSUM2=GXSUM2+G(I)*F(KY-(I-1))
450   CONTINUE
      GO TO 452
451   GXSUM2=0.0
452   GFSUM=0
C CALCULATE TVC ANY NUMBER OF PERIODS FOR ANY Y
800   IF(KYMIN1.LE.0.0)GO TO 801
      TVC(KY)=((C*KYMIN1)+TPENAL+ZEROX+GFSUM+GXSUM1+GXSUM2)
      IF(KY.EQ.NM)GO TO 30
```

```
      TVCDIF=TVC(KY)-TVC(KY+1)
      IF (TVCDIF.LT.0.0)GO TO 30
      TVCSTR=TVC(KY+1)
      KYSTAR(II)=KY
      KYSTR=KY+1
      GO TO 805
801   TVC(1)=(TPENAL+ZEROX+GFSUM+GXSUM1+GXSUM2)
      TVCDIF=TVC(1)-TVC(2)
      IF(TVCDIF.LT.0.0)GO TO 802
      TVCSTR=TVC(2)
      KYSTAR(II)=1
      KYSTR=2
      GO TO 805
802   TVCSTR=TVC(1)
      KYSTAR(II)=0
      KYSTR=1
805   IF(KYMIN1.LT.1.0) GO TO 851
      DO 850 I=1, KYMIN1
850   TVC(I)=0
851   DO 905 I=1,NM
         IF(I.LE.KYSTR) GO TO 901
         F(I)=TVC(I)-C*(I-1))*A
         GO TO 905
901   F(I)=(TVCSTR-C*(I-1))A
905   CONTINUE
```

```
      GO TO 940
      PRINT 907,(G(I),I=1,11),(G(I),I=12,21),(G(I),I=22,31)
907   FORMAT(' ','G(U)=',11F10.7)
      PRINT 915,(TVC(I),I=1,11),(TVC(I),I-12,21),(TVC(I),I=22,31)
915   FORMAT (' ','TVC=',11F10.2)
      PRINT 916,(F(I),I=1,11),(F(I),I=12.21),(F(I),I=22,31)
916   FORMAT(' ','F=',11F10.2)
940   QUARTR(II)=KPERIO
      PRINT 943
943   FORMAT('0',' QTR     N     C     H     P     A
     1PROB     Y*     TVC*')
      PRINT 947,KPERIO,N,C,H,P,A,PROB,KYSTAR(II),TVCSTR
947   FORMAT('0',I3,I8,F12.2F12.2,F10.2,F7.3,F7.2,I7,F12.2)
948   IF(KPERIO.LT.LIM) GO TO 10
      GO TO 1000
950   KY=0
      NTOT=0
      II=1
      DO 955 I=1,LIM
         READ(9,*) N
         PRINT 951
951      FORMAT(' ','PERIOD       N')
         PRINT 952,I,N
952      FORMAT(' ',I4,I9)
         NTOT=NTOT+N
```

```
955   CONTINUE
      TVCSTR=NTOT*PROB*P
      PRINT 956
956   FORMAT('0','C=P CASE   N       C      H      P      A
     1  PROB    Y*       TVC*')
      PRINT 957,N,C,H,P,A,PROB,KY,TVCSTR
957   FORMAT('0',I12,F12.2,F12.2,F10.2,F7.3,F7.2,I7,F12.2)
1000  RETURN
      END
$ENTRY
```

## APPENDIX C

## EIGHT-PERIOD DATA

p = 0.1
A = 0.974

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| C = | 250 | 250 | 250 | 250 | 500 | 500 | 750 | 750 | 1000 | 1000 |
| H = | 0 | 0 | 250 | 250 | 50 | 50 | 150 | 150 | 250 | 250 |
| P = | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 |
| CASE | | | | | | | | | | |
| 1 | 5 | 6 | 2 | 4 | 3 | 5 | 2 | 4 | 0 | 4 |
| 2 | 5 | 6 | 2 | 4 | 3 | 5 | 2 | 4 | 0 | 4 |
| 3 | 4 | 6 | 2 | 4 | 3 | 5 | 2 | 4 | 0 | 4 |
| 4 | 5 | 6 | 2 | 4 | 3 | 5 | 2 | 4 | 0 | 4 |
| 5 | 4 | 6 | 2 | 4 | 2 | 3 | 1 | 4 | 0 | 3 |
| 6 | 5 | 6 | 2 | 4 | 3 | 5 | 2 | 4 | 0 | 4 |
| 7 | 2 | 4 | 1 | 3 | 1 | 3 | 0 | 3 | 0 | 2 |
| 8 | 3 | 4 | 1 | 3 | 1 | 3 | 0 | 3 | 0 | 3 |
| 9 | 3 | 4 | 1 | 3 | 1 | 4 | 0 | 3 | 0 | 3 |
| 10 | 3 | 4 | 1 | 3 | 2 | 4 | 1 | 3 | 0 | 3 |
| 11 | 3 | 5 | 2 | 3 | 2 | 4 | 1 | 3 | 0 | 3 |

p = 0.5

A = 0.974

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| C = | 250 | 250 | 250 | 250 | 500 | 500 | 750 | 750 | 1000 | 1000 |
| H = | 0 | 0 | 250 | 250 | 50 | 50 | 150 | 150 | 250 | 250 |
| P = | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 |
| CASE | | | | | | | | | | |
| 1 | 12 | 13 | 9 | 11 | 10 | 12 | 8 | 11 | 0 | 11 |
| 2 | 12 | 13 | 9 | 11 | 10 | 12 | 8 | 11 | 0 | 11 |
| 3 | 12 | 13 | 9 | 11 | 10 | 12 | 8 | 11 | 0 | 11 |
| 4 | 12 | 13 | 9 | 11 | 10 | 12 | 8 | 11 | 0 | 11 |
| 5 | 12 | 13 | 8 | 11 | 9 | 12 | 7 | 11 | 0 | 11 |
| 6 | 12 | 13 | 9 | 11 | 10 | 12 | 8 | 11 | 0 | 11 |
| 7 | 9 | 11 | 6 | 9 | 7 | 10 | 5 | 9 | 0 | 9 |
| 8 | 10 | 12 | 8 | 10 | 8 | 11 | 7 | 10 | 0 | 10 |
| 9 | 10 | 12 | 8 | 10 | 8 | 11 | 7 | 10 | 0 | 10 |
| 10 | 11 | 12 | 8 | 11 | 9 | 11 | 8 | 11 | 0 | 10 |
| 11 | 12 | 13 | 9 | 11 | 10 | 12 | 8 | 11 | 0 | 11 |

p = 0.9

A = 0.974

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| C = | 250 | 250 | 250 | 250 | 500 | 500 | 750 | 750 | 1000 | 1000 |
| H = | 0 | 0 | 250 | 250 | 50 | 50 | 150 | 150 | 250 | 250 |
| P = | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 |
| CASE | | | | | | | | | | |
| 1 | 15 | 15 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |
| 2 | 15 | 15 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |
| 3 | 15 | 15 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |
| 4 | 15 | 15 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |
| 5 | 15 | 15 | 14 | 15 | 15 | 15 | 13 | 15 | 0 | 15 |
| 6 | 15 | 15 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |
| 7 | 14 | 15 | 13 | 15 | 13 | 15 | 12 | 15 | 0 | 14 |
| 8 | 15 | 15 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |
| 9 | 15 | 15 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |
| 10 | 15 | 15 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |
| 11 | 15 | 15 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |

p = 0.1

A = 1.0

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| C = | 250 | 250 | 250 | 250 | 500 | 500 | 750 | 750 | 1000 | 1000 |
| H = | 0 | 0 | 250 | 250 | 50 | 50 | 150 | 150 | 250 | 250 |
| P = | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 |
| CASE | | | | | | | | | | |
| 1 | 7 | 8 | 2 | 4 | 3 | 5 | 2 | 4 | 0 | 4 |
| 2 | 9 | 11 | 2 | 4 | 3 | 5 | 2 | 4 | 0 | 4 |
| 5 | 7 | 9 | 2 | 4 | 3 | 5 | 1 | 4 | 0 | 4 |
| 8 | 3 | 4 | 1 | 3 | 1 | 3 | 0 | 3 | 0 | 3 |
| 11 | 3 | 5 | 2 | 3 | 2 | 4 | 1 | 3 | 0 | 3 |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |

p = 0.5

A = 1.0

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| C = | 250 | 250 | 250 | 250 | 500 | 500 | 750 | 750 | 1000 | 1000 |
| H = | 0 | 0 | 250 | 250 | 50 | 50 | 150 | 150 | 250 | 250 |
| P = | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 |
| CASE | | | | | | | | | | |
| 1 | 15 | 15 | 9 | 11 | 10 | 12 | 8 | 12 | 0 | 11 |
| 2 | 15 | 15 | 9 | 11 | 10 | 12 | 8 | 12 | 0 | 11 |
| 5 | 29 | 30 | 8 | 11 | 9 | 12 | 7 | 11 | 0 | 11 |
| 8 | 10 | 12 | 7 | 10 | 8 | 11 | 7 | 10 | 0 | 10 |
| 11 | 12 | 13 | 9 | 11 | 10 | 12 | 8 | 12 | 0 | 11 |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |

p = 0.9

A = 1.0

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| C = | 250 | 250 | 250 | 250 | 500 | 500 | 750 | 750 | 1000 | 1000 |
| H = | 0 | 0 | 250 | 250 | 50 | 50 | 150 | 150 | 250 | 250 |
| P = | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 |
| CASE | | | | | | | | | | |
| 1 | 15 | 15 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |
| 2 | 15 | 15 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |
| 5 | 30 | 29 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |
| 8 | 15 | 15 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |
| 11 | 15 | 15 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |

# APPENDIX D

## TWO AND THREE-PERIOD DATA

A = 0.974

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| C = | 250 | 250 | 250 | 250 | 500 | 500 | 750 | 750 | 1000 | 1000 |
| H = | 0 | 0 | 250 | 250 | 50 | 50 | 150 | 150 | 250 | 250 |
| P = | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 |
| CASE/P | | | | | | | | | | |
| 11/0.1 | 3 | 5 | 2 | 4 | 2 | 4 | 1 | 4 | 0 | 3 |
| 11/0.5 | 12 | 13 | 9 | 11 | 10 | 12 | 8 | 11 | 0 | 11 |
| 11/0.9 | 15 | 15 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |
| | | | | | | | | | | |
| 9/0.1 | 3 | 4 | 2 | 4 | 2 | 4 | 1 | 3 | 0 | 3 |
| 9/0.5 | 10 | 12 | 8 | 11 | 9 | 11 | 7 | 11 | 0 | 10 |
| 9/0.0 | 15 | 15 | 14 | 15 | 15 | 15 | 14 | 15 | 0 | 15 |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |

# APPENDIX E

## SINGLE-PERIOD DATA

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| C = | 250 | 250 | 250 | 250 | 500 | 500 | 750 | 750 | 1000 | 1000 |
| H = | 0 | 0 | 250 | 250 | 50 | 50 | 150 | 150 | 250 | 250 |
| P = | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 | 1000 | 10000 |
| p | | | | | | | | | | |
| .1 | 2 | 4 | 2 | 4 | 1 | 4 | 1 | 3 | 0 | 3 |
| .5 | 9 | 11 | 8 | 11 | 7 | 11 | 6 | 10 | 0 | 10 |
| .9 | 14 | 15 | 14 | 15 | 14 | 15 | 13 | 15 | 0 | 15 |

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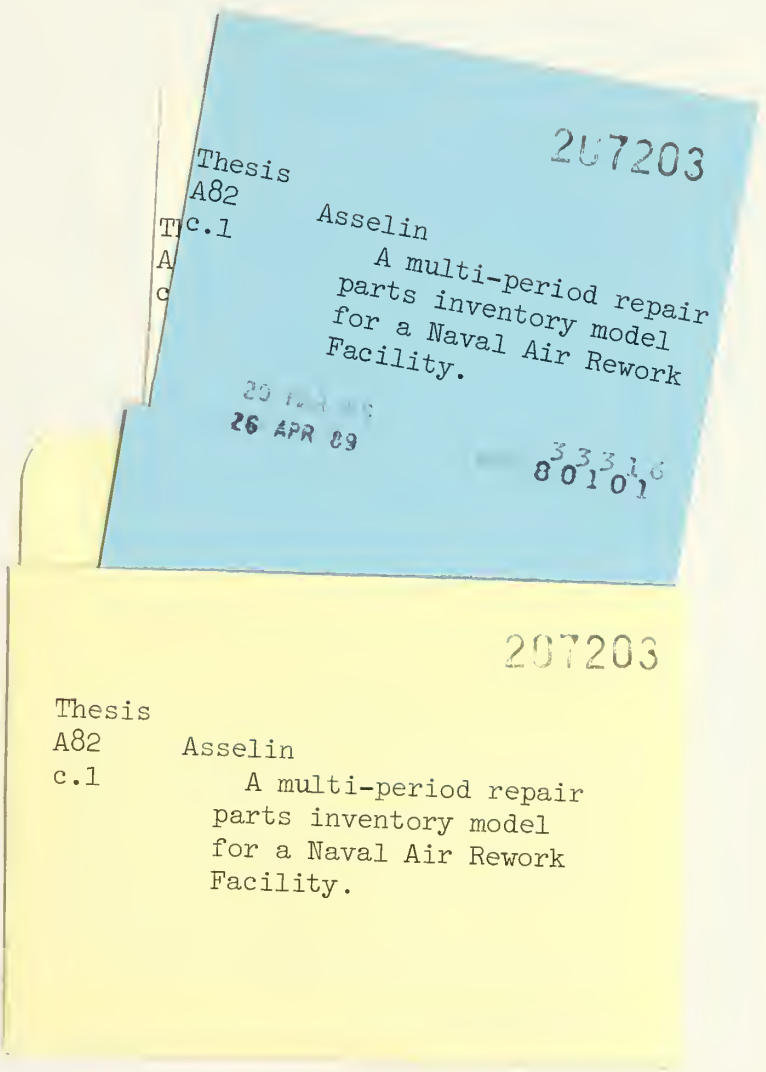
10. Lieutenant Commander John J. Hund, SC, USN 1
U.S.S. IWO JIMA (LPH-2)
FPO, New York 09561

11. Commander Andre S. Asselin, SC, USN 2
6246 Windward Drive
Burke, Virginia 22015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